

---



---

## IT FORMULATES FOR I NUMBER PI

---



---

### THEOREM

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1) * (2^n + 1)} =$$

$$- \pi/2 + \ln(2) + 1$$

---

### DEMONSTRATION

If we take the well-known formula for the number pi

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n * (n+1) * (2^n + 1)} = \pi - 3$$

Preserving denominators is possible to build a  
New formula

$$1/n - 1/(2^n + 1) = (n+1)/(n * (2^n + 1))$$

$$1/(n+1) - 1/(2^n + 1) = n/((n+1) * (2^n + 1))$$

$$(n+1)/(n * (2^n + 1)) + n/((n+1) * (2^n + 1)) =$$

$$((n+1)^2 + n^2)/(n * (n+1) * (2^n + 1)) =$$

$$(2^n + 2^n + 1)/(n * (n+1) * (2^n + 1))$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} * (2^n + 2^n + 1)}{n * (n+1) * (2^n + 1)}$$

This formula is equal to the succes numbers

$$+(1/1 - 1/3) + (1/2 - 1/3)$$

$$-(1/2 - 1/5) - (1/3 - 1/5)$$

$$+(1/3 - 1/7) + (1/4 - 1/7)$$

$$-(1/4 - 1/9) - (1/5 - 1/9)$$

Fractions that occupy the new posts  $2^n$  and  $4^n$  is equal to

$$- 2/3 + 2/5 - 2/7 + 2/9 - 2/11 + 2/13 - 2/15 + 2/17 - \dots$$

Which equals to  $\pi/2 - 2$

The remaining fractions cancel each other except first fraction  $1/1$

Therefore the formula

$$\sum_{n=1}^{\infty} [ (-1)^{(n+1)} * \frac{2*n^2 + 2*n + 1}{n * (n + 1)*(2*n + 1)} ]$$

Equals to  $\pi/2 - 1$

If we divide this formula into two adding

$$\sum_{n=1}^{\infty} [ (-1)^{(n+1)} * ( \frac{2*n^2 + 2*n}{n * (n + 1)*(2*n + 1)} + \frac{1}{n * (n + 1)*(2*n + 1)} ) ]$$

The sum of the terms of adding the second is equal to the known formula for the Number Pi

$$\sum_{n=1}^{\infty} [ \frac{(-1)^{(n+1)}}{n * (n + 1)*(2*n + 1)} ] = \pi - 3$$

Therefore the sum of the terms of adding first

$$\sum_{n=1}^{\infty} [ (-1)^{(n+1)} * \frac{2*n^2 + 2*n}{n * (n + 1)*(2*n + 1)} ]$$

Is equal to the total result less  $(\pi - 3)$

$$(\pi/2 - 1) - (\pi - 3) = -\pi/2 + 2$$

This formula can be divided into two adding

$$\sum_{n=1}^{\infty} [ (-1)^{(n+1)} * ( \frac{2*n^2}{n * (n + 1)*(2*n + 1)} + \frac{2*n}{n * (n + 1)*(2*n + 1)} ) ]$$

The first adding can be written as

$$\sum_{n=1}^{\infty} [ (-1)^{n+1} * \frac{2^n}{(n+1) * (2^n + 1)} ]$$

And dividing by two

$$\sum_{n=1}^{\infty} [ (-1)^{n+1} * \frac{n}{(n+1) * (2^n + 1)} ]$$

If we develop in fractions obtained

$$1/(n+1) - 1/(2^n+1) = n/((n+1) * (2^n+1))$$

Therefore adding this is equal to the succes numbers

$$+(1/2 - 1/3)$$

$$-(1/3 - 1/5)$$

$$+(1/4 - 1/7)$$

$$-(1/5 - 1/9)$$

$$+(1/6 - 1/11)$$

Fractions that occupy positions pairs equals

$$- 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - 1/15 + 1/17 - 1/19 - ...$$

Which equals to  $\pi/4 - 1$

Fractions that are odd is equal to

$$+ 1/2 - 1/3 + 1/4 - 1/5 + 1/6 - 1/7 + 1/8 - ...$$

Which equals - logarithm (2) + 1

Hence the first adding equals

$$\pi/4 - \ln(2)$$

As we have divided by two equals

$$\pi/2 - 2*\ln(2)$$

Therefore the sum of the terms of the second adding

$$\sum_{n=1}^{\infty} [ (-1)^{(n+1)} * \frac{2^n}{n * (n + 1) * (2^n + 1)} ]$$

Is equal to the total result less (PI/2 - 2\*LN(2))

$$(-\text{PI}/2 + 2) - (\text{PI}/2 - 2 * \text{LN}(2)) = -\text{PI} + 2 * \text{LN}(2) + 2$$

Can be written as

$$\sum_{n=1}^{\infty} [ (-1)^{(n+1)} * \frac{2}{(n + 1) * (2^n + 1)} ]$$

And finally if we divide by two you get the final result

$$\sum_{n=1}^{\infty} [ \frac{(-1)^{(n+1)}}{(n + 1) * (2^n + 1)} ] =$$

$$-\text{PI}/2 + \text{LN}(2) + 1$$