
FORMULAS FOR THE NUMBER PI

THEOREM

$$\sum_{n=1}^{\infty} \frac{8^n}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} = \text{PI}/4 + \text{LN}(2)$$

$$\sum_{n=1}^{\infty} \frac{8^n - 4}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} = \text{PI}/4$$

DEMONSTRATION

I will first show the second and then made the first
 Each term can be divided into the following parts

$$+ (1/(4^{n-3}) - 1/(4^{n-2})) + (1/(4^{n-2}) - 1/(4^{n-1}))$$

The first is equal to adding

$$((4^{n-2}) - (4^{n-3})) / ((4^{n-2}) \cdot (4^{n-3}))$$

Which equals

$$1 / ((4^{n-3}) \cdot (4^{n-2}))$$

The second is equal to adding

$$((4^{n-1}) - (4^{n-2})) / ((4^{n-2}) \cdot (4^{n-1}))$$

Which equals

$$1 / ((4^{n-2}) \cdot (4^{n-1}))$$

If the two join totaling

$$1 / ((4^{n-3}) \cdot (4^{n-2})) + 1 / ((4^{n-2}) \cdot (4^{n-1})) =$$

$$((4^{n-1}) + (4^{n-3})) / ((4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})) =$$

$$(8^{n-4}) / ((4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}))$$

Therefore the formula

$$\text{SUM}_{n=1}^{\text{infinity}} \left[\frac{8^n - 4}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} \right]$$

Is equal to the succes numbers

$$+(1/1 - 1/2) + (1/2 - 1/3)$$

$$+(1/5 - 1/6) + (1/6 - 1/7)$$

$$+(1/9 - 1/10) + (1/10 - 1/11)$$

$$+(1/13 - 1/14) + (1/14 - 1/15)$$

Fractions with denominator couple cancel each other
And the remainder is equal to

$$+ 1/1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - 1/15 + \dots$$

Which equals PI/4

Therefore the formula

$$\text{SUM}_{n=1}^{\text{infinity}} \left[\frac{8^n - 4}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} \right]$$

Equals PI/4

DEMONSTRATION OF THE FIRST FORMULA

In a previous message demonstrate formula

$$\text{SUM}_{n=1}^{\text{infinity}} \left[\frac{16^n - 6}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} \right] =$$

$$\text{PI/4} - \text{LN}(2)/2$$

There is a well-known formula for the number pi E. H. Clarke

$$\sum_{n=1}^{\infty} \frac{1}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} =$$

$$\ln(2)/4 - \pi/24$$

Demonstrate that the formula in a previous message can be
Write as

$$\sum_{n=1}^{\infty} \frac{16^n}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} - \frac{6}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} =$$

$$\pi/4 - \ln(2)/2$$

The sum of the terms of the second adding equals

$$\sum_{n=1}^{\infty} \frac{6}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)}$$

That is equal to the formula E. H. Clarke multiplied by 6
Or $(3 \cdot \ln(2))/2 - \pi/4$

Thus the sum of the terms of adding first

$$\sum_{n=1}^{\infty} \frac{16^n}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)}$$

Can be written as

$$\sum_{n=1}^{\infty} \frac{4}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})}$$

Is equal to the total amount made the most of E. H. Clarke
Multiplied by 6

$$\pi/4 - \ln(2)/2 + (3 \cdot \ln(2))/2 - \pi/4 = \ln(2)$$

Therefore the numbers succes

$$4/(1 \cdot 2 \cdot 3) + 4/(5 \cdot 6 \cdot 7) + 4/(9 \cdot 10 \cdot 11) + 4/(13 \cdot 14 \cdot 15) + 4/(17 \cdot 18 \cdot 19) + \dots$$

Equals LN(2)

The formula to demonstrate that principle can be written as

$$\sum_{n=1}^{\infty} \frac{8^n}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} - \frac{4}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} = \text{PI}/4$$

The sum of the terms of the second adding

$$\sum_{n=1}^{\infty} \frac{4}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})}$$

Is equal to the succession of numbers that we have seen for
The logarithm(2)

Thus the sum of the terms of adding first
Is equal to the overall result more LN(2)

$$\text{PI}/4 + \text{LN}(2)$$

$$\sum_{n=1}^{\infty} \frac{8^n}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})} = \text{PI}/4 + \text{LN}(2)$$