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## FORMULA FOR THE NUMBER PI

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### THEOREM

$$\sum_{n=1}^{\infty} \frac{1}{(4^{n-3}) \cdot (4^n)} = \frac{\pi}{24} + \frac{\ln(2)}{4}$$

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### DEMONSTRATION

First take the formula that was equal to Logarithm(2)

$$\sum_{n=1}^{\infty} \frac{2 \cdot (4^{n-1}) \cdot (4^{n-2}) + 2}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} = \ln(2)$$

This formula can be shown as follows

Each term of the formula can be divided into two parts

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Part I

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$$+ (1/(4^{n-3}) - 1/(4^{n-2})) + (1/(4^{n-2}) - 1/(4^{n-1}))$$

This in turn two adding up  
The first is equal to adding

$$1/((4^{n-3}) \cdot (4^{n-2}))$$

The second is equal to adding

$$1/((4^{n-2}) \cdot (4^{n-1}))$$

If the two join totaling

$$\begin{aligned} & 1/((4^{n-3}) \cdot (4^{n-2})) + 1/((4^{n-2}) \cdot (4^{n-1})) = \\ & ((4^{n-1}) + (4^{n-3})) / ((4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})) = \\ & (8^{n-4}) / ((4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1})) \end{aligned}$$

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Part Two

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$$+ (1/(4^{*n-1}) - 1/(4^{*n-2})) + (1/(4^{*n-1}) - 1/(4^{*n}))$$

The first is equal to adding

$$- 1/((4^{*n-1})*(4^{*n-2}))$$

The second is equal to adding

$$+ 1/((4^{*n-1})*(4^{*n}))$$

If the two join totaling

$$- 1/((4^{*n-1})*(4^{*n-2})) + 1/((4^{*n-1})*(4^{*n})) =$$

$$(- (4^{*n}) + (4^{*n-2}))/((4^{*n-1})*(4^{*n-2})*(4^{*n})) =$$

$$- 2/((4^{*n-1})*(4^{*n-2})*(4^{*n}))$$

And finally, if the two sides join in which divide  
Each term

$$(8^{*n-4})/((4^{*n-3})*(4^{*n-2})*(4^{*n-1})) - 2/((4^{*n-2})*(4^{*n-1})*(4^{*n})) =$$

$$((4^{*n})*(8^{*n-4}) - 2*(4^{*n-3}))/((4^{*n-3})*(4^{*n-2})*(4^{*n-1})*(4^{*n}))$$

Numerator can be written as

$$(4^{*n})*(8^{*n-4}) - (8^{*n-6}) =$$

$$(4^{*n})*(8^{*n-4}) - ((8^{*n-4}) - 2) =$$

$$(4^{*n-1})*(8^{*n-4}) + 2 =$$

$$2*(4^{*n-1})*(4^{*n-2}) + 2$$

You get the formula final

$$\text{infinity} \quad 2*(4^{*n-1})*(4^{*n-2}) + 2 \\ \text{SUM} [ \frac{\text{-----}}{\text{-----}} ] \\ n=1 \quad (4^{*n-3})*(4^{*n-2})*(4^{*n-1})*(4^{*n})$$

Therefore this formula is equal to the succes numbers

$$+ (1/1 - 1/2) + (1/3 - 1/4)$$

$$+ (1/5 - 1/6) + (1/7 - 1/8)$$

$$+ (1/9 - 1/10) + (1/11 - 1/12)$$

Which equals LN(2)

If we divide it into two adding up

$$\text{SUM}_{n=1}^{\text{infinity}} \left[ \frac{2 \cdot (4^{n-1}) \cdot (4^{n-2})}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} + \frac{2}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} \right] = \text{LN}(2)$$

The sum of the terms of the second adding

$$\text{SUM}_{n=1}^{\text{infinity}} \left[ \frac{2}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} \right]$$

Is equal to the formula E. H. Clarke

$$\text{SUM}_{n=1}^{\text{infinity}} \left[ \frac{1}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} \right] =$$

$$- \text{PI}/24 + \text{LN}(2)/4$$

Multiplied by two

$$- \text{PI}/12 + \text{LN}(2)/2$$

Thus the sum of the terms of adding first

$$\text{SUM}_{n=1}^{\text{infinity}} \left[ \frac{2 \cdot (4^{n-1}) \cdot (4^{n-2})}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} \right]$$

Is equal to the total result unless the formula E. H. Clarke

Multiplied by 2

$$\text{LN}(2) - (- \text{PI}/12 + \text{LN}(2)/2) = \text{PI}/12 + \text{LN}(2)/2$$

$$\text{SUM}_{n=1}^{\text{infinity}} \left[ \frac{2 \cdot (4^{n-1}) \cdot (4^{n-2})}{(4^{n-3}) \cdot (4^{n-2}) \cdot (4^{n-1}) \cdot (4^n)} \right] = \text{PI}/12 + \text{LN}(2)/2$$

Can be written as

$$\sum_{n=1}^{\infty} \frac{2}{(4^n-3)(4^n)} = \frac{\pi}{12} + \frac{\ln(2)}{2}$$

And finally, if we divide it by two obtained the formula  
End

$$\sum_{n=1}^{\infty} \frac{1}{(4^n-3)(4^n)} = \frac{\pi}{24} + \frac{\ln(2)}{4}$$